Accurate Modeling of Thin Conducting Layers in FDTD

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***Abstract*— In recent papers the surface impedance condition is proposed to incorporate the effect of high conductivity of a material. This condition only takes the reflection at materials into account; the transmission through very thin layers is not considered. In this letter, we introduce a similar approach to model both reflection and transmission when confronted with a thin layer of good, but not perfectly, conducting material.**

***Index Terms*—Conductors, electromagnetic fields, FDTD methods.**

I. Introduction

In recent papers, the surface impedance boundary condition is studied and implemented for the finite-difference time-domain (FDTD) method [1], [2]. This surface impedance relates the electric and magnetic fields at the surface of the body. As such, reflection from a good conductor is accurately modeled. Other methods for modeling of conducting scatterers are proposed in [3]. However, transmission through a thin layer of conducting material has never been considered.

In another paper, thin conducting layers are modeled with a thin layer formalism [4]. Yet this formalism demands that the thickness of the layer is small compared to the skin depth of the material, since the electric field within the layer is again assumed to vary linearly.

In this letter, we model a thin layer of conducting material, incorporating the skin depth and the transmission into the formalism. With this new formalism, reflection and transmission for conductors with a thickness of the order of the skin depth can be taken into account.

II. The Transmission Line Approximation

We will consider a thin material layer with material properties ϵ, μ, and σ. Examples of such layers are metallized foils or cloths, radoms, water layers, or vapor deposits on plastics. The layer itself has a thickness d which is smaller than the FDTD cell size Δ. This layer is pictured in Fig. 1. The fields inside the layer are of no interest, so we will only take the fields at the edge of the layer into account.

We assume that the following conditions apply.

- The conductivity of the layer is so high that the waves inside the layer can be assumed to propagate only perpendicularly to the layer surfaces (this means σ is large compared to $\omega\epsilon$).

Manuscript received September 19, 1997.

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Publisher Item Identifier S 1051-8207(98)01462-7.

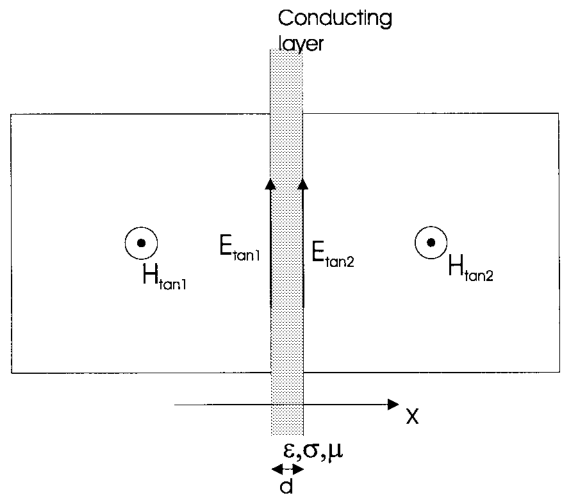

Fig. 1. Layout of the conducting layer and neighboring fields.

- The local curvature radius of the layer is large compared to the layer thickness d.
- The layer thickness is small compared to the transversal dimensions of the layer.

In this case, it can be assumed that the electric field $\boldsymbol{E}$ inside the layer locally satisfies the one-dimensional wave equation

$$\frac{\partial^2 \boldsymbol{E}}{\partial x^2} + k^2 \boldsymbol{E} = 0 \tag{1}$$

with $k = \omega\sqrt{(\epsilon + (\sigma/j\omega))\mu} \approx \sqrt{-j\sigma\mu\omega}$.

We can now relate the tangential electric fields at each side of the layer to the magnetic fields at the same locations:

$$\begin{pmatrix} \boldsymbol{E}_{\tan 1} \\ \boldsymbol{E}_{\tan 2} \end{pmatrix} = \begin{pmatrix} Z_{11} & -Z_{12} \\ Z_{12} & -Z_{11} \end{pmatrix} \begin{pmatrix} \boldsymbol{n} \times \boldsymbol{H}_{\tan 1} \\ \boldsymbol{n} \times \boldsymbol{H}_{\tan 2} \end{pmatrix}. \tag{2}$$

According to transmission line theory Z_{11} and Z_{12} can be written as

$$Z_{11} = -\frac{j\omega\mu}{k}\,\frac{1}{\tan(kd)} \tag{3}$$

$$Z_{12} = -\frac{j\omega\mu}{k}\,\frac{1}{\sin(kd)}. \tag{4}$$

These equations have been used widely to treat the thin lossy layers in the frequency domain [5]. However, in this letter we transform the frequency-domain equations to the time domain, making it possible to use the same method in FDTD.

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III. Transformation to the FDTD Technique

FDTD is a time domain technique, while (2) is formulated in the frequency domain. If we would apply the inverse Fourier transform to (2), it would result in a convolution, which implies that all previous values of the magnetic fields neighboring a conducting surface would have to be remembered. This would be unacceptable.

However, we can rewrite (3) as

$$Z_{11} = -\frac{j\omega\mu}{k} \frac{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} (kd)^{2n}}{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (kd)^{2n+1}} \tag{5}$$

and the same can be done with Z_{12}. For simplicity, we will only show our approach using Z_{11}, Z_{12} is treated in exactly the same way.

If we assume that the layer is thinner than five skin depths at the highest frequency, we only have to account for the first seven terms in the summations. In the case where we have a thicker layer, one could consider modeling the reflection by using the surface impedance boundary conditions and ignoring the transmission. On the other hand, more terms may be used to approximate thicker planes; the method remains unaltered.

Hence (5) can be truncated and rewritten as

$$Z_{11} = -\frac{1}{\sigma d} \frac{\sum_{n=0}^{N} \frac{(j\omega\sigma\mu d^2)^n}{(2n)!}}{\sum_{n=0}^{N} \frac{(j\omega\sigma\mu d^2)^n}{(2n+1)!}}. \tag{6}$$

Now we apply the substitution $j\omega' = j\omega\sigma\mu d^2$. This removes the dependency of the medium parameters from (6). It is possible to rewrite (6) as

$$Z_{11} = -\frac{1}{\sigma d} \left(\sum_{n=0}^{m} \frac{A_{11,n}}{j\omega' + B_n} + \sum_{k=0}^{l} \frac{C_{11,k} j\omega' + D_{11,k}}{(j\omega')^2 + E_k j\omega' + F_k} \right) \tag{7}$$

with $m + 2l = N$. How the coefficients $A_{11,n}$, B_n, $C_{11,n}$, $D_{11,n}$, E_n, and F_n are obtained is discussed later. The same approach can again be used for Z_{12}. Using (7) and a similar expression for Z_{12} we can rewrite the electric field components of (2) as a summation:

$$\begin{aligned} \boldsymbol{E}_{\tan 1} = &-\frac{1}{\sigma d} \left(\sum_{n=0}^{m} \frac{A_{11,n}}{j\omega' + B_n} + \sum_{k=0}^{l} \frac{C_{11,k} j\omega' + D_{11,k}}{(j\omega')^2 + E_k j\omega' + F_k} \right) \boldsymbol{n} \times \boldsymbol{H}_{\tan 1} \\ &+ \frac{1}{\sigma d} \left(\sum_{n=0}^{m} \frac{A_{12,n}}{j\omega' + B_n} + \sum_{k=0}^{l} \frac{C_{12,k} j\omega' + D_{12,k}}{(j\omega')^2 + E_k j\omega' + F_k} \right) \boldsymbol{n} \times \boldsymbol{H}_{\tan 2}. \end{aligned} \tag{8}$$

TABLE I
Coefficients Needed for the Seven ($N = 7$) Term Expansion of Z_{11} and Z_{12}

i	$A_{11,i}$	$A_{12,i}$	B_i	
1	19.739	-19.765	9.8696	
2	-129.94	137.91	41.165	
3	102.20	-443.42	50.491	
i	$C_{11,i}$	$D_{11,i}$	$C_{12,i}$	$D_{12,i}$
1	1.1071	297.24	1627.6	-19139
2	12.9402	755.07	569.14	48185
i	E_i	F_i		
1	7.9892	13092		
2	137.914	41.1657		

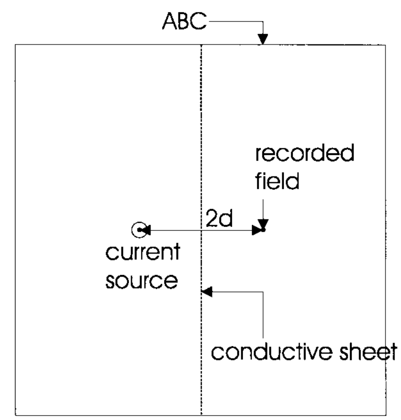

Fig. 2. Layout of the 2-D mesh.

Each term in this equation can be written as $\boldsymbol{X}_i$, and $\boldsymbol{X}_i$ relates to $\boldsymbol{H}_{\tan 1}$ or $\boldsymbol{H}_{\tan 2}$ through a first or second order differential equation. These differential equations can be discretized in the usual FDTD way [6].

Three problems arise.

- First of all we have to find the right values for $A_n, B_n, \cdots$. This can be done by using a partial fraction routine. The advantage of the method is that this has to be done only once, since the dependency of the medium parameters is included in ω'. Once the values are found we can transform ω' back to ω and adapt the values of $A_n, B_n, \cdots$ accordingly, before transforming the equation to the time domain. When we use a seven term expansion we find the values presented in Table I.
- Another problem is finding the correct values for $\boldsymbol{H}_{\tan}$, since in the discretized FDTD space the values for $\boldsymbol{H}_{\tan}$ are not known at the edge of the conducting layer, as is shown in Fig. 1. An often used approximation is to assume that the value of the magnetic field next to the layer is the correct value. Other approximations, involving interpolation are possible.
- The third problem concerns the insertion of the formalism in the FDTD mesh. A discretized version of (8) and a similar equation for $\boldsymbol{E}_{\tan 2}$ can be used to update the tangential electric fields. Both electric fields are

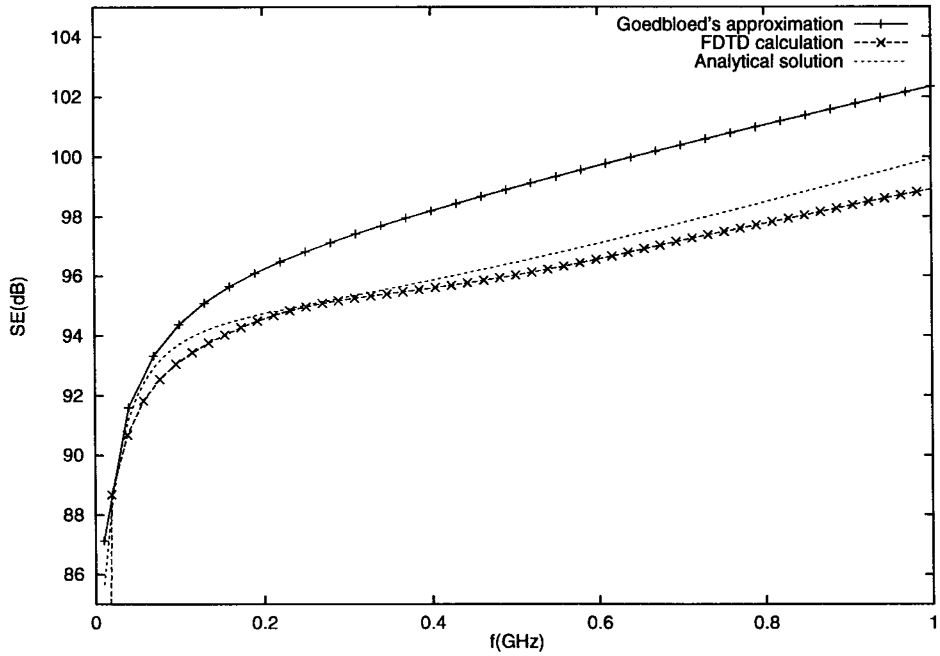

Fig. 3. Shielding efficiency of a thin sheet in the near field of a current line source.

positioned at the same location in the mesh since the conducting sheet is normally very thin compared to the spacestep. However, to update the neighboring $\boldsymbol{H}_{\tan 1}$ and $\boldsymbol{H}_{\tan 2}$ fields the corresponding electric field on the same side is used, i.e., $\boldsymbol{E}_{\tan 1}$ ($\boldsymbol{E}_{\tan 2}$) is used to update $\boldsymbol{H}_{\tan 1}$ ($\boldsymbol{H}_{\tan 2}$).

IV. Shielding Efficiency of a Conducting Sheet

To test the formalism in a two-dimensional (2-D) FDTD program, we calculate the shielding efficiency of an infinite thin copper plate in the near field of a current line source. We use the 2-D TM polarization. The layout of the mesh is shown in Fig. 2.

The advantage of this simple example is that we can, by using a spatial decomposition of the field, calculate the exact solution to the shielding problem. As such the field at the point of interest is given by

$$\boldsymbol{E}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -j\omega\mu_0 e^{-2\Gamma_1 d} \tag{9}$$

$$\frac{2\Gamma_2}{(\Gamma_1+\Gamma_2)^2 e^{\Gamma_2\Delta} - (\Gamma_1-\Gamma_2)^2 e^{-\Gamma_2\Delta}}\, dk_y \tag{10}$$

with $\Gamma_1 = \sqrt{k_y^2 - k_1^2}$, $\Gamma_2 = \sqrt{k_y^2 - k_2^2}$, where k_1, and k_2 are the wavenumbers in vacuum and within the sheet, d is the distance between the source or the recorder and the sheet, and Δ is the thickness of the sheet (in this case 5 μm). When the shield is not present we get

$$\boldsymbol{E}(\omega) = -\frac{k_1 R_0}{4}\, \mathcal{H}_0^{(2)}(k_1|\boldsymbol{r}-\boldsymbol{r}_0|)\boldsymbol{u}_z. \tag{11}$$

Divide both fields to obtain the shielding efficiency. Fig. 3 shows the result of the analytical approach, compared to the result obtained by the FDTD formalism. The FDTD shielding efficiency differs only slightly (1 dB at most) with the exact analytical solution. According to J. J. Goedbloed another approximation may be used [7], neglecting the multiple reflections inside the sheet. This approximation differs about two decibels with the exact solution.

V. Conclusions

We have developed a formalism, which can be used to model thin sheets of good conducting materials in FDTD. Both the reflection and the transmission of the sheet are included as well as the effect of the skin depth. The formalism is accurate as long as the sheet remains thinner than five skin depths at the highest frequency.

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